## IIT Networks and Optimization Seminar

# OPTIMIZATION MODELING FOR TRADEOFF ANALYSIS OF HIGHWAY INVESTMENT ALTERNATIVES


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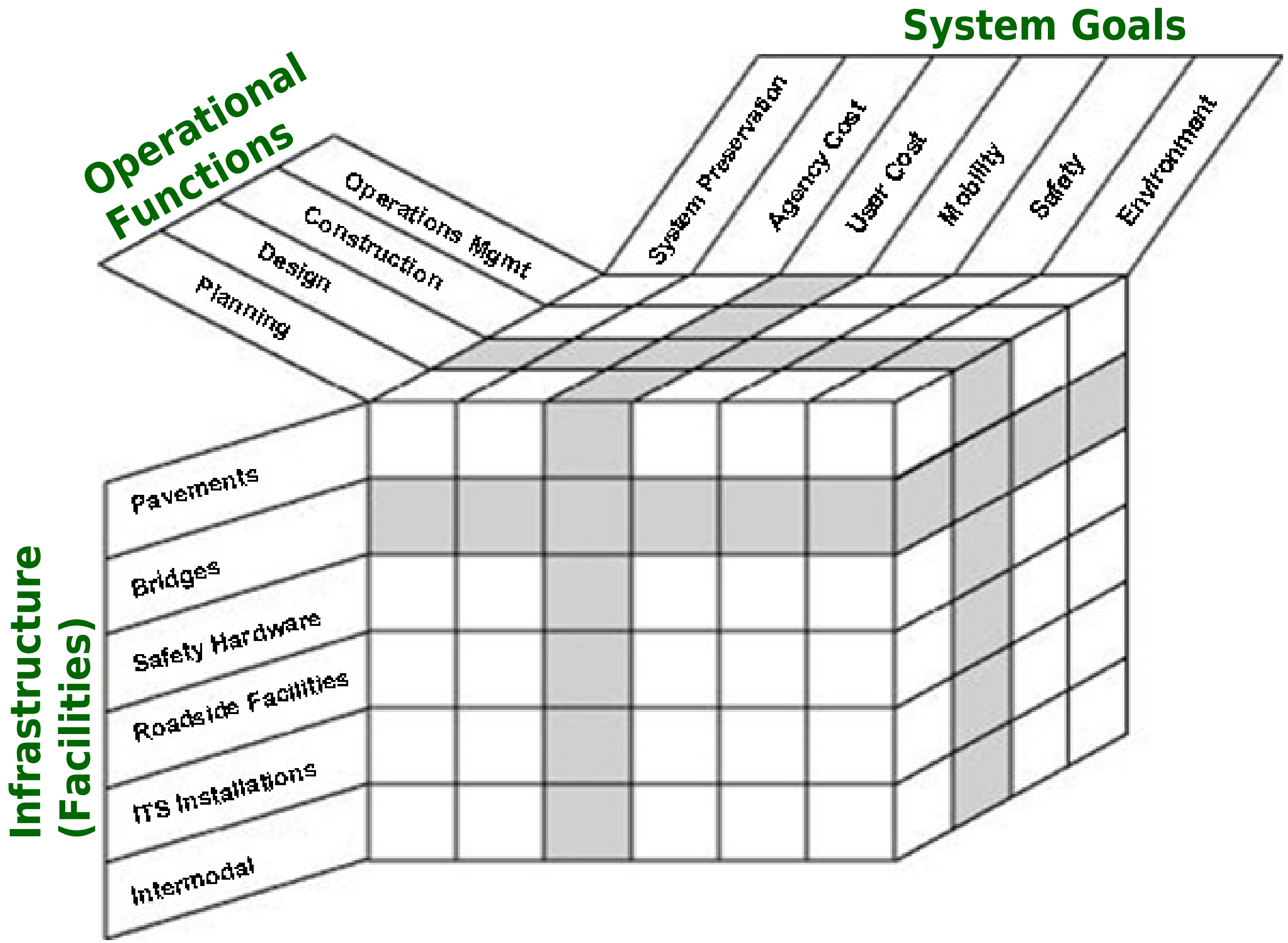
# Contents of This Presentation

- Introduction to Transportation Asset Management
- Optimization Formulations for Project Selection
- A Computational Study
- Work in Progress
- Concluding Remarks

# Dimensions of a Highway Transportation System



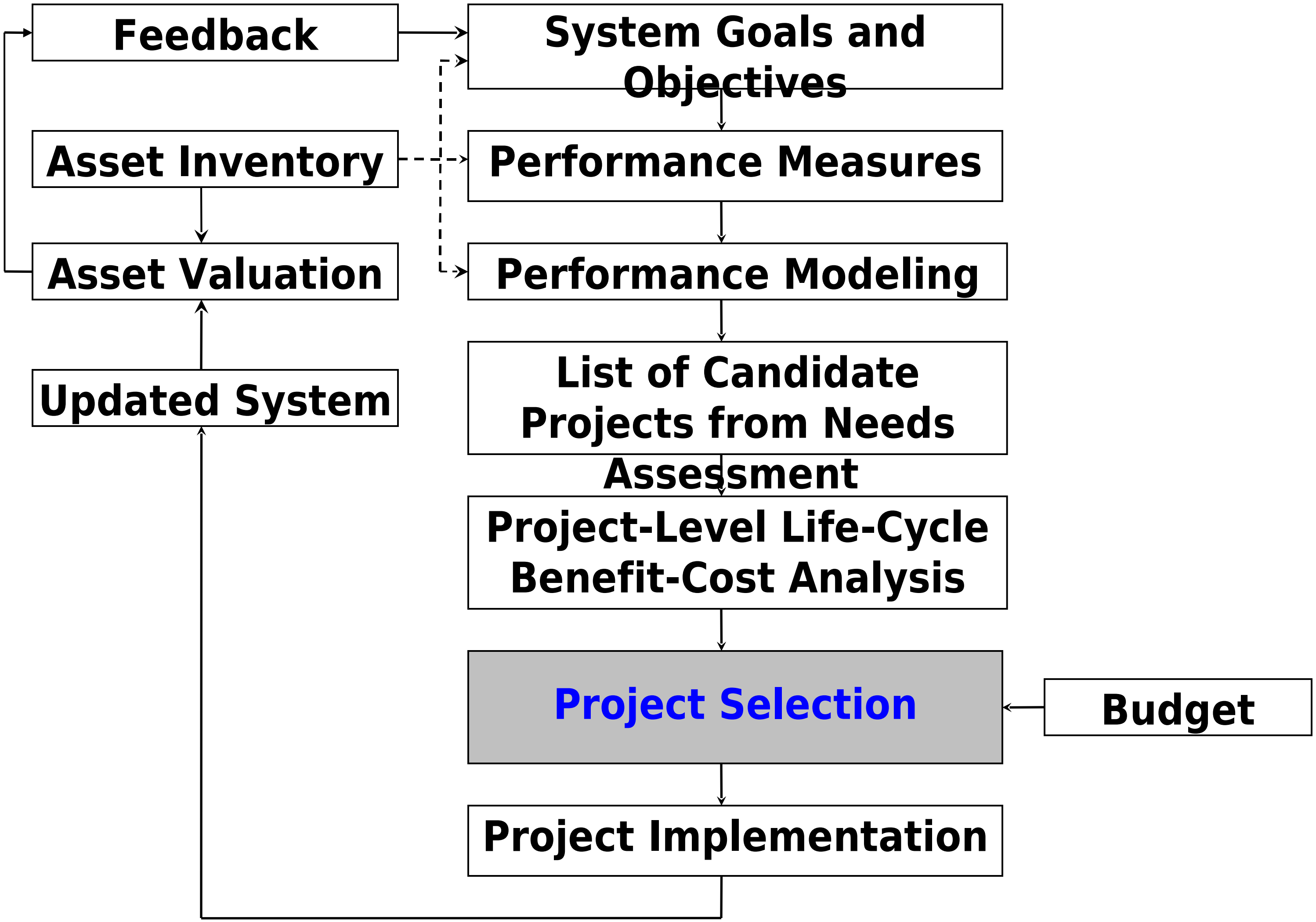
# Existing Analytical Tools for Investment Decision-Making

- Pavement Management Systems
- Bridge Management Systems
- Maintenance Management Systems
- Safety Management Systems
- Congestion Management Systems

# Need for Overall Highway Asset Management

- Interdependency of System Components
- Increasing System Demand
- Budget Pressure
- Accountability Requirements
- Technological Advancements

# Highway Asset Management System Components



# Optimization Formulation for Systemwide Highway Project Selection

- **As the 0-1 Multi-Choice Multidimensional Knapsack Problem**
  - Multi-choice corresponds to multiple categories of budgets designated for different highway management programs
  - Multi-dimension refers to a multiyear project implementation period, and
  - The objective is to select a subset from all economically feasible candidate projects to achieve maximized total benefits under various constraints.
- **Basic Model**

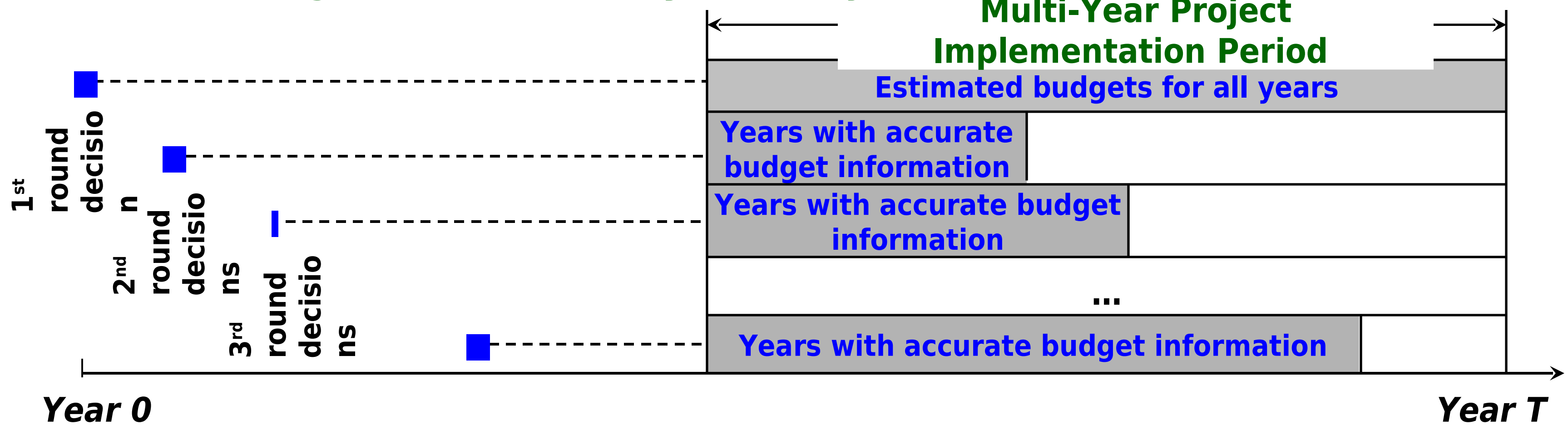
Maximize $A^T.X$

Subject to $C_{kt}^T.X \le B_{kt}$

where A is the vector of benefits of *N* projects, $A = [a_1, a_2, \ldots, a_N]^T$, X is the decision vector for all decision variables, $X = [x_1, x_2, \ldots, x_N]^T$, $C_{kt}$ is the vector of costs of *N* projects using budget

# Addressing Budget Uncertainty in Project Selection

## Issues of Budget Uncertainty in Project Selection




## Using Recourse Decisions to Address Budget Uncertainty

| Year | 1 to $t_1$ | $t_1$+1 to $t_2$ | ... | $t_{(L-2)}$+1 to $t_{(L-1)}$ | $t_{(L-1)}$+1 to $t_L$ | $t_L$+1 to $t_{(L+1)}$ | ... |
|---|---|---|---|---|---|---|---|
| Budget | 1 possibility | $s_2$ possibilities | ... | $s_{(L-1)}$ possibilities | $s_L$ possibilities | $s_{(L+1)}$ possibilities | ... |

**Stage 1:** Deterministic (Initially estimated budgets)

**Stage 2:** Deterministic | Stochastic *($p_2 = s_2.s_3...s_{L-1}.s_L.s_{L+1}$... combinations)*

**...** ...

**Stage *L*-1:** Deterministic | Stochastic *($p_{L-1} = s_{L-1}.s_L.s_{L+1}$... combinations)*

**Stage *L*:** Deterministic | Stochastic *($p_L = s_L.s_{L+1}$... combinations)*

**Stage** Deterministic | Stochastic *(p*

# A Stochastic Model with Ω-stage Budget Recourse Decisions

$$\text{Maximize} \quad A^T.X_1 + E_{\xi 2}[Q_2(X_2(p), \xi_2)] + \ldots + E_{\xi\Omega}[Q_\Omega(X_\Omega(p), \xi_\Omega)] \qquad (1)$$

*Stage 1:* Subject to $C_{kt}^T.X_1 \le E(B_{kt}^1)$ (2)

*Stage 2:* $E_{\xi 2}[Q_2(X_2(p), \xi_2)] = \max\{A^T.X_2(p) \mid B_{kt}^2(p) = E(B_{kt}^2)\}$ (3)

Subject to $C_{kt}^T.X_2(p) \le B_{kt}^2(p)$ (4)

$X_1 + X_2(p) \le 1$ (5)

…

*Stage L:* $E_{\xi L}[Q_L(X_L(p), \xi_L)] = \max\{A^T.X_L(p) \mid B_{kt}^L(p) = E(B_{kt}^L)\}$ (6) (7)

Subject to $C_{kt}^T.X_L(p) \le B_{kt}^L(p)$ (8)

$X_1 + X_2(p) + \ldots + X_L(p) \le 1$

…

*Stage Ω:* $E_{\xi\Omega}[Q_\Omega(X_\Omega(p), \xi_\Omega)] = \max\{A^T.X_\Omega(p) \mid B_{kt}^\Omega(p) = E(B_{kt}^\Omega)\}$ (9) (10)

Subject to $C_{kt}^T.X_\Omega(p) \le B_{kt}^\Omega(p)$ (11)

$X_1 + X_2(p) + \ldots + X_L(p) + \ldots + X_\Omega(p) \le 1$

where A is the vector of benefits of $N$ projects, $A = [a_1, a_2, \ldots, a_N]^T$, $C_{kt}$ is the vector of costs of $N$ projects using budget from management program $k$ in year $t$, $C_{kt} = [c_{1kt}, c_{2kt}, \ldots, c_{Nkt}]^T$, $X_L(p)$ is the decision vector using budget $B_{kt}^L(p)$ at stage $L$, $X_L(p) = [x_1, x_2, \ldots, x_N]^T$, $a_i$ is benefits of project $i$, $i = 1, 2, \ldots, N$, $c_{ikt}$ is costs of project $i$ using budgets from management program $k$ in year $t$, $x_i$ is the decision variable for project $i$, $\xi_L$ is randomness associated with budgets at stage $L$ and decision space, $Q(X_L(p), \xi_L)$ is the recourse function at stage $L$, $E_{\xi 2}[Q(X_L(p), \xi_L)]$ is the mathematical expectation of the recourse function at stage $L$, $B^L_{kt}(p)$ is the $p^{th}$ possibility of budget for management program $k$ in year $t$ at stage $L$, $p(B^L_{kt}(p))$ is the probability of having budget scenario

$$E(B_{kt}^L) = \sum_{p=1}^{p_L} [P(B_{kt}^L(p)) \cdot B_{kt}^L(p)]$$

# Budget for Stage *L* Computation

- **Criterion to determine Budget for Stage L Computation**
  - For yearly constrained budget scenario: Minimize $\Delta B^L(p) = \sum_{k=1}^{K}\sum_{t=1}^{M}\left[\left(B_{kt}^L(p) - E\left(B_{kt}^L\right)\right)^2\right]$
  - For cumulative budget scenario: Minimize $\Delta B^L(p) = \sum_{k=1}^{K}\left[\left(\sum_{t=1}^{M} B_{kt}^L(p) - \sum_{t=1}^{M} E\left(B_{kt}^L\right)\right)^2\right]$

    where $E\left(B_{kt}^L\right) = \sum_{p=1}\left[P\left\langle B_{kt}^L(p)\right\rangle \cdot B_{kt}^L(p)\right]$

- **An Example**

Budget Possibility 1 (10% Chance)

| t | No change | | | 25% Lower |
|---|---|---|---|---|
| k | 1 | 2 | 3 | 4 |
| 1 | 1 | 100 | 120 | 75 |
| 2 | 100 | 100 | 1.2 | 0.75 |
| 3 | 1 | 100 | 120 | 75 |
| 4 | 1 | 1 | 1.2 | 75 |
| 5 | 100 | 100 | 120 | 75 |

6,006

Budget Possibility 2 (25% Chance)

| t | No change | | | No change |
|---|---|---|---|---|
| k | 1 | 2 | 3 | 4 |
| 1 | 1 | 100 | 120 | 100 |
| 2 | 100 | 100 | 1.2 | 1 |
| 3 | 1 | 100 | 120 | 100 |
| 4 | 1 | 1.2 | 1.2 | 100 |
| 5 | 100 | 100 | 120 | 100 |

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Budget Possibility 3 (65% Chance)

| t | No change | | | 25% Higher |
|---|---|---|---|---|
| k | 1 | 2 | 3 | 4 |
| 1 | 1 | 100 | 120 | 125 |
| 2 | 100 | 100 | 1.2 | 1.25 |
| 3 | 1 | 100 | 120 | 125 |
| 4 | 1 | 1 | 1.2 | 125 |
| 5 | 100 | 100 | 120 | 125 |

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| t | Expected Budget | | | |
|---|---|---|---|---|
| k | 1 | 2 | 3 | 4 |
| 1 | 1 | 100 | 120 | 114 |
| 2 | 100 | 100 | 1 | 1 |
| 3 | 1 | 100 | 120 | 114 |
| 4 | 1 | 1 | 1 | 114 |
| 5 | 100 | 100 | 120 | 114 |

# Enhanced Stochastic Model

- **Incorporate Segment-Based Project Implementation Option**
  - Tie-ins of multiple projects within one highway segment or across multiple highway segments for actual implementation
  - Benefits of all constituent projects of a segment-based “project group” added together
  - The constituent projects may request budgets from different programs in multiple years
  - The size of the decision vector in the stochastic model is reduced.
- **Incorporate Corridor-Based Project Implementation Option**
  - As an extension of segment-based project implementation option, the tie-ins of multiple projects within one or more highway segments is extended to a freeway corridor or a major urban arterial corridor
  - Benefits of all constituent projects of a corridor-based “grand project group” combined
  - The constituent projects may request budgets from different programs in multiple years
  - The size of the decision vector in the stochastic model is further reduced.



- **Incorporate Deferment-Based Project Implementation Option**

# Theorem of Lagrange Multipliers

- **Redefined the optimization model for Stage L**

  **Objective Max $z(Y_L) = A^T.Y_L$**

  **Subject to $C_{kt}^T.Y_L \le B_{kt}^L$, where $Y_L$ is stage *L* decision vector with 0/1 integer elements.**

- **For non-negative Lagrange Multipliers, $\lambda_{kt}$, the Lagrangian relaxation of the model can be written as**

  **Objective** $z_{LR}(\lambda_{kt}) = \max \left\{ A^T.Y_L + \sum_{k=1}^{K}\sum_{t=1}^{M}\left[\lambda_{kt}.\left(B_{kt}^L - C_{kt}^T.Y_L\right)\right]\right\} = \max \left\{ \left(A^T - \sum_{k=1}^{K}\sum_{t=1}^{M}\left(\lambda_{kt}.C_{kt}^T\right)\right)Y_L + \sum_{k=1}^{K}\sum_{t=1}^{M}\left(\lambda_{kt}.B_{kt}^L\right)\right\}$

  **Subject to $Y_L$ with 0/1 integer elements.**

- **The unconstrained solution to** $z'_{LR}(\lambda_{kt}) = \max \left(A^T - \sum_{k=1}^{K}\sum_{t=1}^{M}\left(\lambda_{kt}.C_{kt}^T\right)\right)Y_L$ **is**

  $$Y_L^* = \begin{cases} 1, \text{if } \left(A^T - \sum_{k=1}^{K}\sum_{t=1}^{M}\left(\lambda_{kt}.C_{kt}^L\right)\right) > 0 \\ 0, \text{otherwise} \end{cases}$$

- **The solution algorithm for the original optimization model needs to focus on determining Lagrange Multipliers $\lambda_{kt}$ to satisfy the following conditions:**

  **a) $Y_L^*$ Feasible to Constraint $C_{kt}^T.Y_L^* \le B_{kt}^L$**

  **where** $$Y_L^* = \begin{cases} 1, \text{if } \left(A^T - \sum_{k=1}^{K}\sum_{t=1}^{M}\left(\lambda_{kt}.C_{kt}^L\right)\right) > 0 \\ 0, \text{otherwise} \end{cases}$$

  **b)** $\sum_{k=1}^{K}\sum_{t=1}^{M}\left[\lambda_{kt}\left(B_{kt}^L - C_{kt}^T.Y_L\right)\right] = 0$ **to maintain optimality of original optimization model.**

# Proposed Algorithm for ***Stage L*** Computations

- **Step 0** ***(Initialization and Normalize)***
  - Determine budget $B_{kt}^{L}(p)$ for Stage $L$ such that $\Delta B^{L}(p)$ = minimum { $B_{kt}^{L}(1)$, $B_{kt}^{L}(2)$, …, $B_{kt}^{L}(p_L)$}
  - Select all projects and sore projects by benefits ($A_i$) in descending order
  - Normalize contract costs and budget for each (k, t):

$$c'_{ikt} = c_{ikt} / B_{kt}^{L}(p),\ B_{kt}^{L}(p) = 1 \text{ and } C_{kt} = \sum_{i=1}^{n} c'_{kt}$$

- **Step 1** ***(Determine the Most Violated Constraint k, t)***
  - Set $C'_{kt}$ = maximum $\{C_{kt}\}$ for all k, t
- **Step 2** ***(Compute the Increase of Lagrange Multiplier Value $\lambda_{kt}$)***

$$\theta_i = \frac{A_i - \sum_{k=1}^{K}\sum_{t=1}^{M}(\lambda_{kt} \cdot c_{ikt})}{\sum_{k=1}^{K}\sum_{t=1}^{M}\left(c_{ikt} \cdot \frac{C_{kt}}{\min\{C_{kt}\}}\right)} \text{ for all } k, t \text{ or } \infty \text{, otherwis}$$

- **Step 3** ***(Increase $\lambda_{kt}$*** **by** ***and Reset $X_i$ the Value Zero)***
  - Let $\lambda_{kt} = \lambda_{kt} + \theta'_i \cdot \left(\frac{C_{kt}}{C'_{kt}}\right)$ and $C_{kt} = C_{kt} - c'_{ikt}$ for all $k$, $t$
  - Remove project i and reset decision variable $x_i = 0$
  - If $C_{kt} \leq 1$ for all $k$, $t$, go to Step 4. Otherwise, go to Step 1.
- **Step 4** ***(Improve Solution)***
  - Check whether the projects with zero-variable values can have the value one without violating the constraints $C_{kt} \leq 1$.

# Proposed Algorithm for Stage *L* Operations (Con't)

- **Step 5 *(Further Improve Solution with Budget Carryover)***

  **A small amount of budget might be left after project selection and it could be carried over to the immediate following year one year at a time to repeat Steps 1 to 4 to further improve the solution.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Before | $B_{k1}^{L}(p)$ | $B_{k2}^{L}(p)$ | ... | $B_{k,t-1}^{L}(p)$ | $B_{kt}^{L}(p)$ | $B_{k,\,t+1}^{L}(p)$ | ... | $B_{kM}^{L}(p)$ |
| After | 0 | 0 | ... | 0 | 0 | $B_{k,\,t+1}^{L}(p)$ + $\Delta B_{kt}^{L}(p)$ | ... | $B_{kM}^{L}(p)$ |

*One-period budget carryover for remaining budget from year t to year t+1:*

**Increase budget $B_{k,t+1}^{L}(p)$ by $\Delta B_{kt}^{L}(p) = B_{kt}^{L}(p) - C_{kt}$ and this leaves $B_{kt}^{L}(p) = 0$ after budget carryover.**

**- Hold solution for the preceding years from *1* to *t***

**- Re-optimize for the remaining years from *t*+1 to M**

**- Repeat until reaching the last year M.**

# Computational Complexity of the Proposed Algorithm

- **Steps 1-4: Computational complexity is $O(M.N^2)$**
- **Step 5: Budget carryover requires M iterations**
- **Ω-Stage recourses needs at most M interactions**

**This gives an overall complexity of $O(M^3N^2)$. Since M<<N, the algorithm remains a complexity of $O(N^2)$.**

# A Computational Study for Model Application

- **Candidate Project Data** - Preparation
  - **Eleven-year data on 7,380 candidate projects proposed for Indiana state highway programming during 1996-2006 were used to apply the proposed heuristic approach for systemwide project selection**
  - **Examples of estimated project-level life-cycle benefits:**

| Project No. | Let Year | Lanes | Length (Miles) | AADT | Work Type | Project Cost | Project Benefit Items (%) | | | | | Total Benefits |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | AC | VOC | Mobility | Safety | Env. | |
| 12021 | 2000 | 4 | 0.11 | 69,200 | Bridge widening | 2,291,000 | 4 | 22 | 1 | 55 | 19 | 11,703,264 |
| 12040 | 2000 | 4 | 0.50 | 32,630 | Pavement resurfacing | 4,620,000 | 2 | 33 | 1 | 37 | 27 | 6,365,844 |
| 12077 | 2000 | 2 | 2.06 | 3,170 | Pavement resurfacing | 3,000,000 | 3 | 27 | 1 | 46 | 23 | 15,545,501 |
| 12158 | 1999 | 2 | 3.70 | 16,770 | Added travel lanes | 750,000 | 2 | 30 | 7 | 34 | 26 | 4,806,134 |
| 21749 | 1998 | 2 | 13.63 | 4,190 | Pavement resurfacing | 11,573,000 | | | | 100 | | 63,943,225 |
| 21825 | 1996 | 4 | 2.53 | 11,150 | Pavement rehabilitation | 151,000 | 10 | 32 | 1 | 31 | 27 | 1,505,738 |
| 21931 | 1996 | 4 | 0.78 | 2,664 | Rigid pavement replace | 196,000 | 52 | 20 | 18 | 5 | 5 | 736,046 |
| 21944 | 1996 | 2 | 9.46 | 1,100 | Pavement rehabilitation | 131,000 | | | | 100 | | 353,545 |
| 22026 | 1996 | 2 | 0.15 | 8,291 | Bridge widening | 108,000 | 13 | 28 | 4 | 30 | 26 | 254,516 |
| 22044 | 1996 | 2 | 1.10 | 13,994 | Pavement resurfacing | 2,757,000 | | 26 | 24 | 28 | 22 | 5,702,627 |
| ... | ... | ... | ... | ... | ... | ... | | | | | | ... |

- **Budget Data**
  - **The annual average budgets designated for new construction, pavement preservation, bridge preservation, maintenance, safety improvements, roadside improvements, ITS installations, and miscenaneous programs were approximately 700 million dollars with 4 percent increment per year**
  - **The initial budget estimates were updated three times, providing 4-stage budget recourse decisions.**
- **Considerations of Project Implementation Options**
  - **Segment-based project implementation option: selecting projects by roadway segment**
  - **Corridor-based project implementation option: selecting projects in corridors I-64, I-65, I-69, I-70, I-74, I-80, I-90, and I-94**

# Computational Study Results

## - Comparison of Total Benefits and Matching Rates of Selected Projects

- **Comparison of Total Benefits of Selected Projects**

| Total Benefits (in 1990, Billion Dollars) | | Project Benefits by Highway System Goal | | | | | Total |
|---|---|---|---|---|---|---|---|
| Budget | Project Implementation Option | Agency Cost | VOC | Mobility | Safety | Environment | |
| Deterministic | Segment-based | 9.78 | 4.78 | 3.46 | 15.46 | 4.23 | 37.7 |
| | Corridor-based | 9.34 | 4.3 | 3.35 | 15.14 | 4.19 | 36.8 |
| | Deferment-based | 9.19 | 4.99 | 3.27 | 15.51 | 4.38 | 37.3 |
| Stochastic | Segment-based | 9.87 | 4.86 | 3.52 | 15.66 | 4.30 | 38.2 |
| | Corridor-based | 9.44 | 4.74 | 3.34 | 15.23 | 4.20 | 37.0 |
| | Deferment-based | 9.27 | 5.03 | 3.29 | 15.59 | 4.42 | 37.6 |
| Deterministic | Average | 9.87 | 4.72 | 3.27 | 15.29 | 4.17 | 37.3 |
| Stochastic | | 9.96 | 4.77 | 3.30 | 15.42 | 4.22 | 37.7 |
| Average | Segment-based | 10.27 | 4.64 | 3.36 | 15.71 | 4.11 | 38.1 |
| | Corridor-based | 9.98 | 4.51 | 3.20 | 14.89 | 3.99 | 36.6 |
| | Deferment-based | 9.50 | 5.10 | 3.29 | 15.47 | 4.50 | 37.9 |

- **Comparison of Consistency Matching Rates of Selected Projects**

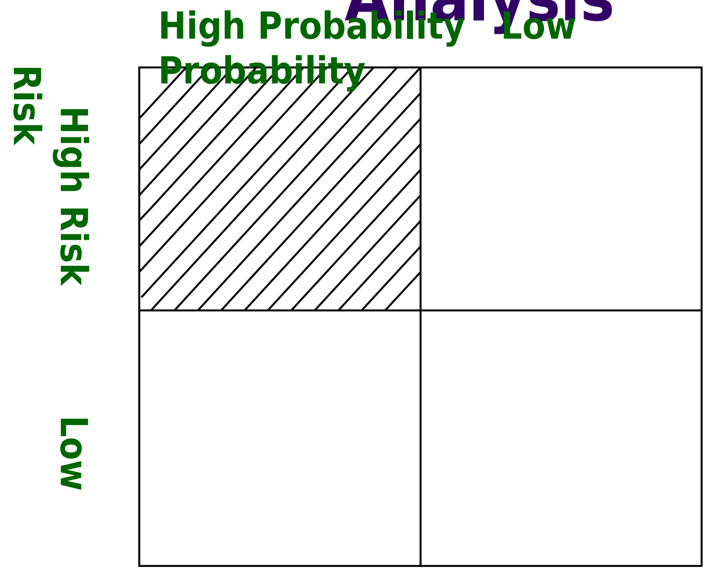
| Comparison Method | | Average Number of Projects Selected | Match with Indiana DOT Authorization | |
|---|---|---|---|---|
| Budget | Project Implementation Option | | Number | Percent |
| Deterministic | Segment-based | 6,016 | 5,050 | 79.6% |
| | Corridor-based | 5,964 | 4,955 | 78.1% |
| | Deferment-based | 6,038 | 5,064 | 79.9% |
| Stochastic | Segment-based | 6,023 | 5,059 | 79.8% |
| | Corridor-based | 6,015 | 5,004 | 78.9% |
| | Deferment-based | 6,024 | 5,051 | 79.7% |
| Deterministic budget | Average | 6,006 | 5,023 | 79.2% |
| Stochastic budget | | 6,021 | 5,038 | 79.5% |
| Average | Segment-based | 6,020 | 5,055 | 79.7% |
| | Corridor-based | 5,990 | 4,980 | 78.5% |
| | Deferment-based | 6,031 | 5,058 | 79.8% |
| Projects Authorized by Indiana DOT | | 6,341 | | |
| Projects Matched for All Project Section Strategies | | | 4,656 | 73.4% |

# Needed Model Enhancements

- The proposed stochastic model addressing budget constraints by program category and by year, project tie-ins, and budget uncertainty is discussed.

- Model enhancements are needed for:
  - Adding chance constraints for expected infrastructure conditions and system operations service levels after project implementation
  - Incorporating constraints for maximum allowable risks in the benefits of interdependent projects that would facilitate tradeoff analysis across different types of assets. This will help answer the following critical questions:
    - What happens if there is an across the board "x" percent decrease in both pavement and bridge investment levels?
    - What happens if funding is increased for the bridge

# Addressing Risks of Project Benefits in Trade-off Analysis




- **Difference between risk and uncertainty**
  - **Risk involves objective probabilities and measurable quantities**
  - **Uncertainty involves subjective probabilities and immeasurable quantities**
- **Financial analysts and engineers have long dealt with the problems of managing, mitigating, and minimizing risk. Among the techniques used are mean-variance analysis, Value at Risk (VaR) and Stochastic Dominance**
- **Selecting projects for transportation asset management is similar to selecting stocks for a portfolio. Instead of stocks, we have highway projects. We will primarily limit our**

# Augmenting the Stochastic Model into Two-Phase Optimization

- **Phase I Optimization: Find Minimum of Risks of Project Benefits**
  - **Markowitz mean-variance model formulation**

    **Min** $$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \operatorname{cov}(b_i, b_j)$$

    **Subject to** $$\sum_{i=1}^{n} x_i \le B = 100\%, \; x_i \ge 0, \text{and } E(b_i) \ge B_i$$

**where $x_i$ is the proportion of our budget in dollar that are invested in project *i*, $b_i$ is the benefits of project *i*, $B_i$ is the threshold benefits of project *i*, B is budget constraint, and *i* = 1, 2, …, n.**

- **Phase II Optimization: Use Optimal Value of the Objective Function from Phase I as Upper Bound Constraint of Risks of Project Benefits added to the Proposed Stochastic Model.**

# Project Benefits, Costs, and Covariance in the Markowitz Model

- **Proportion of Budget to be Used by a Project**

| Project | Benefits | Costs | Proportion of Obtainable Budget |
|---|---|---|---|
| 1 | $b_1$ | $C_1$ | $X_1 = C_1/B$ |
| 2 | $b_2$ | $C_2$ | $X_2 = C_2/B$ |
| 3 | $b_3$ | $C_3$ | $X_3 = C3/B$ |
| ... | ... | ... | ... |
| N | $b_N$ | $C_N$ | $X_N = C_N/B$ |

$$\sum_{i=1}^{N} C_i >> B$$

$$\sum_{i=1}^{N} X_i \le B = 1$$

- **Covariance of Benefits for Each Pair of Projects**

$$COV(b_i, b_j) = E(b_i, b_j) - E(b_i)E(b_j) = \sum_{S=1}^{3}\sum_{T=1}^{3} b_{i,S} b_{j,T} P(b_{i,S}, b_{j,T}) - [\sum_{S=1}^{3} b_{i,S} P(b_{i,S})][\sum_{T=1}^{3} b_{j,T} P(b_{j,T})]$$

| | | $P_i$ | | | |
|---|---|---|---|---|---|
| | | $b_{i,L}$ | $b_{i,M}$ | $b_{i,H}$ | |
| $P_j$ | $b_{j,L}$ | $P(b_{iL}, b_{jL})$ | $P(b_{iM}, b_{jL})$ | $P(b_{iH}, b_{jL})$ | $P(b_{j,L})$ |
| | $b_{j,M}$ | $P(b_{iL}, b_{jM})$ | $P(b_{iM}, b_{jM})$ | $P(b_{iH}, b_{jM})$ | $P(b_{j,M})$ |
| | $b_{j,H}$ | $P(b_{iL}, b_{jH})$ | $P(b_{iM}, b_{jH})$ | $P(b_{iH}, b_{jH})$ | $P(b_{j,H})$ |
| | | $P(b_{i,L})$ | $P(b_{i,M})$ | $P(b_{i,H})$ | |

# Wolfe's LP Formulation for Solving the Markowitz Model

- **Markowitz mean-variance model can be re-written in its general form:**

  **Objective** Min $z(x) = -cx + (1/2)x^TQx$

  **Subject to** $Ax \geq b,\ x \geq 0.$

  **where c = coefficient vector of the decision vector x, $x = [x_1, x_2, \ldots, x_N]^T$, Q = positive definite matrix for the coefficients of the quadratic terms, A = vector of expected benefits of *N* projects, $A = [a_1, a_2, \ldots, a_N]^T$, b = threshold benefits of *N* projects, $b = [b_1, b_2, \ldots, b_N]^T$.**

- **As all variables $x_1, x_2, \ldots, x_N$ are nonnegative, the Wolfe's method could be adopted for solving a LP formulation derived from the Markowitz mean-variance model as follows:**

  **Objective:** $\min w = a_1 + a_2 + \ldots + a_k$

  **Subject to** $Qx - e + A^Ty = c^T$

  $Ax - e' = b$

  $x \geq 0.$



**where $a_1, a_2, \ldots, a_k$ = Non-negative artificial variables, e, e' =**

# The Wolfe's Modified Simplex Algorithm

- **Step 1: Modify the constraints so that the right-hand side of each constraint is non-negative. This requires that each constraint with a negative right-hand side be multiplied through by -1**

- **Step 2: Identify each constraint that is now an "=" or "≥" constraint**

- **Step 3: Cover each inequality constraint to the standard form. If constraint *i* is a "≤" constraint, add a slack variable $s_i$. If constraint *i* is a "≥" constraint, add an excessive variable $e_i$**

- **Step 4: For each "=" or "≥" constraint identified in Step 2, add an artificial variable $a_k$**

- **Step 5: Solve for the LP by satisfying the complementary slackness requirements: ye'= 0 and ex = 0**

  **If the optimal value w > 0, the LP has no feasible solution. The solution x to which w = 0 is the optimal solution to the original Markowitz mean- variance model.**

# Concluding Remarks

- **An improved stochastic model, along with an efficient heuristic algorithm, is introduced to address budget uncertainty and project implementation option issues in systemwide highway project selection**

- **Computational study reveals that the stochastic model is able to determine the best project implementation option aimed to achieve the highest overall return on investments**

- **The stochastic model needs to be further enhanced as two-phase optimization by addressing risks of project benefits to rigorously carry out cross-asset trade-off analysis**

- **The Markowitz mean-variance model could be employed to find the upper bound of the**

# Bio-Sketch of Zongzhi Li

- **Education**
  - Chang'an University (B.E.)
  - Purdue University
    - M.S.C.E. and Ph.D. in Transportation (Advisor: Kumares C. Sinha, U.S. NAE Member)
    - M.S.I.E. in Optimization (Advisor: Thomas L. Morin)
- **Professional Experience**
  - Two World Bank Financed Highway Projects
  - From 2006, nine major research projects funded by ASCE, FHWA, Illinois DOT, Indiana DOT, U of Wisc MRUTC/CFIRE, Purdue JTRP, and Galvin Congestion Initiative (over $1.8 million grants)
- **Research Interests**
  - Transportation systems analysis, evaluation, and asset management
  - Statistical and econometric methods for transportation infrastructure performance modeling and safety analysis
  - Optimization, and risk and uncertainty modeling for transportation infrastructure systems and dynamic traffic networks.